OPTIMAL SITING AND SIZING OF DISTRIBUTED GENERATION IN RADIAL DISTRIBUTION NETWORKS

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Abstract: Recently, the power sector has witnessed significant changes due to the introduction of smart-grid technology and the incremental use of distributed generation (DG). Presence of DG on the distribution system creates a number of problems related to stability, safety, security and reliability of the power system. DG allocation alters the voltages, short circuit currents, power flow, losses and other related parameters. Whether the impact of the DG is positive or negative on the system depends on the size and location of the distributed generation. This paper focuses on testing different sensitivity indices and using effective methods for the optimal location and sizing of the DG by reducing power losses and improving voltage profile. The work has been tested on an IEEE 33 bus radial distribution system.

Keywords: RDS, DG, 33-bus, siting, sizing

1. Introduction

Distributed generation is basically a modern approach in the power sector. Certain researchers have defined ‘DG’ by rating units, whereas others have defined ‘DG’ in terms of the technology that has been used for generation. Distributed generation also has several different names, depending on the region. For example, in certain parts of North America, the term Dispersed Generation is common, whereas in South America, the term ‘Embedded Generation’ has been fabricated. In Europe and certain Asian countries, ‘Decentralized Generation’ is used.

2. Literature review

Research is underway on DG allocation since two decades. Some of the important works have been mentioned in this section. A new voltage stability index for finding the most sensitive node with large voltage drop was given in [1]. Composite load modeling was also considered for analyzing voltage stability [2]. Load flow techniques based on the backward forward sweeps were properly evaluated under different R/X ratios, loading conditions and sub-station voltage levels. Static load modeling effects on the convergence of the algorithms were also noted [3]. Hereford Ranch Algorithm (HRA) was employed to obtain best DG size and location that reduced the distribution power losses [4]. The optimal capacities corresponding to each network were evaluated using a direct equation obtained from the sensitivity equation [5].

A deterministic methodology based on SQP algorithm was developed to find the optimal location and size of DG [6]. Genetic Algorithm (GA) was used for optimally allocating a DG for loss reduction and voltage profile improvement [7]. A novel solution was developed for unbalanced three-phase networks based on the loop-analysis method [8]. A loss sensitivity factor based method for the DG allocation, with equivalent current injection was used for the determination of the optimal location and size of DG [9]. Artificial bee colony (ABC) algorithm was used to find the optimal location, size and operating power factor for DG in order to minimize the net real power loss for the system [10]. Optimal locations were then decided depending on loss sensitivity of buses with respect to active power injection at different nodes [12].

A Hybrid Genetic Algorithm - Particle Swarm Optimization (HGAPSO) based algorithm focusing on optimal DG allocation in distribution system was given showing significant performance improvement [13]. Graphical information of network and power flow equations was developed to meet the needs of distribution automation [14]. A method based on basic electric circuit theorems helped in obtaining nodes beyond each lateral [15]. A PSO and sensitivity analysis based method was presented for optimal DG sizing and placement for loss and total harmonic distortion reduction and voltage profile improvement in
distribution networks [16]. A sufficient sensitivity test for the first problem was suggested determining the optimal DG size was obtained using a new heuristic curve-fitted technique that minimized the search-space by selecting fewer DG-tests [17]. Fuzzy logic was used for optimal siting and sizing of DG [18]. A simple method based on voltage sensitivity index (VSI) analysis was introduced [19]. The ICA algorithm was used to find size and location of the DGs and the capacitors [20]. An overview of the models and methods applied to the ODGP problem, analysing and classifying current and future research trends in this topic [21].

3. Problem formulation

This section shows the development of a mathematical model for objective function and related constraints for radial distribution system in case of DG allocation.

3.1 Objective function:

The optimal siting and sizing of DG problem to minimize the total real power loss and voltage profile improvement can be expressed as [36]:

Min \( P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} (\alpha_{ij}(P_iP_j + Q_iQ_j) + \beta_{ij}(Q_iP_j - P_iQ_j)) \)  
(3.1)

where

\( \alpha_{ij} = \frac{r_{ij}}{V_iV_j} \cos(\delta_i - \delta_j) \)

\( \beta_{ij} = \frac{r_{ij}}{V_iV_j} \sin(\delta_i - \delta_j) \)

\( Z_{ij} = r_{ij} + jx_{ij} \)

where \( Z_{ij} \) is the impedance of line between bus i and bus j, \( r_{ij} \) is the resistance of line between bus i and bus j, \( x_{ij} \) is the reactance of line between bus i and bus j, \( V_i \) is the voltage magnitude at bus i, \( V_j \) is the voltage magnitude at bus j.

3.2 Constraints:

The objective function in (3.1) is subjected to the following constraints.

(a) Bus voltage limits:

It is well known that a small variation in the bus voltage affects the flow of reactive power whereas active power practically does not change much. Further, the operating voltage at each bus must be in prescribed limits as shown.

\( V_{\text{min}} \leq V_i \leq V_{\text{max}} \quad i \in \{1,2,3, ..., N_b\} \)  
(3.2)

where, \( V_{\text{min}}, V_{\text{max}} \) = minimum and maximum voltage limits of \( i^{th} \) node, \( V_i \) = voltage at \( i^{th} \) node and \( N_b \) = number of buses.

(b) Feeder capacity limits:

Power flow in each branch should be less than or equal to its maximum limit as given below.

\( |I_i| \leq I_{i\text{max}} \quad i \in \{1,2,3, ..., N_b\} \)  
(3.3)

where, \( I_{i\text{max}} \) = maximum current capacity of \( i^{th} \) branch and \( I_i \) = current in \( i^{th} \) branch.

(c) Power flow equations:

Total real and reactive power generation must be equivalent to the sum of total real power losses and total real component of load.

\( \sum P_{\text{Gen}} = P_L + \sum P_{\text{Load}} \)  
(3.5)

\( \sum Q_{\text{Gen}} = Q_L + \sum Q_{\text{Load}} \)  
(3.6)

where,

\( \Sigma P_{\text{Gen}} \) = Total real power generation,

\( \Sigma Q_{\text{Gen}} \) = Total reactive power generation.

\( P_L \) = Total real power loss.

\( Q_L \) = Total reactive power loss.

\( \Sigma P_{\text{Load}} \) = Total real component of load.

\( \Sigma Q_{\text{Load}} \) = Total reactive load.

3.3 Load Flow of Distribution Network:

Power flow in a radial distribution system can be done by backward sweep and forward sweep method (BFSM).

3.3.1 Branch from bus Fi(i) to bus Ti(i) of a distribution network.
Figure 3.1 represents the $i^{th}$ branch of a distribution network which is connected between bus $F(i)$ and bus $T(i)$ where $P^F_i$ and $Q^F_i$ are real and reactive injected power at bus $F(i)$ respectively, $P^L_i$ and $Q^L_i$ are real and reactive load power at bus $F(i)$ respectively. $P^T_i$ and $Q^T_i$ are real and reactive injected power at bus $T(i)$ respectively. $P^F_T$ and $Q^F_T$ are real and reactive load power at bus $T(i)$ respectively. $P^L_T$ and $Q^L_T$ are real and reactive power flow from bus $T(i)$ respectively. $R_i$ and $X_i$ are series resistance and reactance of the $i^{th}$ branch respectively. $P_i$ and $Q_i$ are real and reactive power between bus $F(i)$ and point ‘A’, respectively. $P_T$ and $Q_T$ are real and reactive power flow just after point ‘A’, respectively. $I_i$ is current in the $i^{th}$ branch between point ‘A’ and ‘B’. $Y_{ci}$ is shunt admittance of the $i^{th}$ branch. $V_{F(i)}$ and $V_{T(i)}$ are voltages at bus $F(i)$ and bus $T(i)$ respectively. Therefore we have:

\[ I^2_i = \frac{(P_i^2 + Q_i^2)}{V_{F(i)}^2} \]  
\[ P_i^* = P^F_i + P^L_i \]  
\[ Q_i^* = Q^F_i + Q^L_i + \frac{V_{T(i)}^2}{2} \]  
\[ P_i = P_i^* + \left( \frac{(P_i^2 + Q_i^2)}{V_{T(i)}^2} \right) \times R_i \]  
\[ Q_i = Q_i^* + \frac{(P_i^2 + Q_i^2)}{V_{T(i)}^2} \times X_i \]  
\[ V_{T0} = V_{F0} \left\{ \frac{P_i - Q_i}{V_{T0}} \right\} \times (R_i + jX_i), \quad \text{Or} \]  
\[ V_{T0} = V_{F0} \left\{ \frac{(P_i R_i + Q_i X_i)}{V_{F0}} \right\} - \frac{1}{2} \left\{ \frac{(P_i X_i - Q_i R_i)}{V_{F0}} \right\} \]  

Let, $V_{T0} = V_{T0} + 0$, then

\[ V_{T0}^2 = \left[ \frac{(P_i R_i + Q_i X_i)}{V_{F0}} \right]^2 + \left[ \frac{(P_i X_i - Q_i R_i)}{V_{F0}} \right]^2 \]  

After simplifying we get

\[ V_{T0}^2 = V_{F0}^2 - 2(P_i R_i + Q_i X_i) + \frac{(P_i^2 + Q_i^2)(R_i^2 + X_i^2)}{V_{F0}^2} \]  

\[ \delta_{T0} - \delta_{F0} = \tan^{-1} \left( \frac{(P_i X_i - Q_i R_i)}{V_{T0}^2 - (P_i R_i + Q_i X_i)} \right) \]  

In BFSM method of load flow, the following steps are involved:

**(a) Branch numbering:**

The process of numbering of branches in a network requires the construction of a network tree. The tree is constructed showing several layers and it starts at the substation or root bus where the source is connected. The swing or slack bus of the network is considered as the root bus. All branches that are connected to the root bus form the first layer. The second layer consists of all branches that are connected to the receiving end bus of the branches in the first layer and so on. All branches of the network should be present in the tree and they should appear only once. The node nearer to the source is called as the parent node and the other node is known as the child node. Initially, a flat voltage start is considered (1 p.u. at all buses).

**Figure 3.2 Layer formations in BFSM load flow**

**(b) Backward Sweep:**

The aim of the backward sweep is to find the power flow through each branch in the tree in a backward direction by taking the previous iteration voltages at each node. Line flows are calculated using (3.8) to (3.13) starting from last layer towards first layer. The backward direction means the equations are first applied to the last branch of the tree and then proceeded in reverse direction until the root branch is reached. During backward sweep, voltage values are held fixed and updated power flows are taken backwards along the feeder using backward flow.
(c) Forward Sweep:
The aim of the forward sweep is to calculate the voltages at each bus starting from the root node. The root node voltage is set as 1.0 per unit and other node voltages are calculated using (3.14) and (3.15). Therefore, \( V_{T(i)} \) and \( \delta_{T(i)} \) are calculated starting from first layer moving towards last layer. The power flow in each branch is taken as a constant at the value obtained during backward substitution. Thus, using the power flows calculated during backward substitution, the values of voltages are calculated which are used for calculating the power flows by backward substitution in the next iteration.

3.4 Proposed methods:
Various methods are available for sizing and siting of DG in radial distribution networks. Out of these techniques two techniques are very good and efficient which will be used in this work and results for these methods will be compared. Optimal operating power factor will also be calculated in addition to the siting and sizing of DG.

3.4.1 Loss Sensitivity Analysis
Loss sensitivity analysis method is mainly used to solve the capacitor allocation problem. Its application in DG allocation is new in this field and has been reported in [1]. The real power loss in the system is given by an exact loss formula. The sensitivity factor of real power loss with respect to real power injection is obtained by differentiating exact loss formula with respect to real power injection at bus \( P_i \) which is given by:

\[
\alpha_i = \frac{\partial P_{li}}{\partial P_i} = 2 \sum_{j=1}^{N} (\alpha_{ij} P_j - \beta_{ij} Q_j) \tag{3.16}
\]

Sensitivity indices are evaluated at all nodes, by using the values obtained at base case load flows i.e. without DG. The buses are ranked in descending order of the values of sensitivity indices to form a priority list. The total power loss against injected power is a parabolic function and at minimum of losses, the rate of change of real power loss with respect to real power injection becomes zero.

\[
\alpha_i = \frac{\partial P_{li}}{\partial P_i} = 2 \sum_{j=1}^{N} (\alpha_{ij} P_j - \beta_{ij} Q_j) = 0 \tag{3.17}
\]

which gives,

\[
P_i = \frac{1}{\alpha_i} \left[ \beta_{ii} Q_i + \sum_{j=1}^{N} \left( \alpha_{ij} P_j - \beta_{ij} Q_j \right) \right] \tag{3.18}
\]

where \( P_i \) is the real power injection at node \( i \), and is the difference between real power generation and real power demand at that node.

\[
P_i = P_{DG_i} - P_{DI} \tag{3.19}
\]

where \( P_{DG_i} \) is the real power injection from DG placed at node \( i \), \( P_{DI} \) is the load demand at node \( i \), combining (3.18) & (3.19) we get

\[
P_{DG_i} = P_{DI} + \frac{1}{\alpha_i} \left[ \beta_{ii} Q_i - \sum_{j=1}^{N} \left( \alpha_{ij} P_j - \beta_{ij} Q_j \right) \right] \tag{3.20}
\]

The above equation defines the size of the DG at which the losses will be minimized. By arranging the list in ascending order, the bus stood in the top is ranked as the first priority for DG and further the process is repeated by placing the optimal size of DG at that particular location which generates the next location of DG. The process is said to be terminated when it determines the same location again and again.

3.4.2 Voltage Sensitivity Index Method
This is another method which will be used for reducing the search space. In this case each bus is penetrated at a time, by a DG of 20% size of the maximum feeder loading capacity. After putting DG at each node its voltage sensitivity indices can be calculated by Eq. (3.21). When DG is connected at bus ‘\( i \)’, voltage sensitivity index for bus \( i \) is given by:

\[
BVSI = \sqrt{\frac{\sum_{k=1}^{N} (1-V_k)^2}{N}} \tag{3.21}
\]

where \( V_k \) is the voltage at kth node and \( N \) is the number of nodes. The node with the least BVSI will be chosen for DG placement. The algorithm for DG location and sizing can be given as:

**Step 1:** Run load flow for base case.

**Step 2:** Find the Bus voltage sensitivity indices at each node using Eq. (3.21) by penetrate the 20% of DG value at respective node and rank the sensitivities of all nodes in ascending order to form priority list.
Step 3: Select the bus with lowest priority and place DG at that bus.

Step 4: Change the size of DG in small steps and calculate power loss for each by running load flow.

Step 5: Store the size of DG that gives minimum loss.

Step 6: Compare the loss with the previous solution. If loss is less than previous solution, store this new solution and discard previous solution.

Step 7: Repeat Step 4 to Step 6 for all buses in the priority list.

Step 8: End

3.5 Test System

An IEEE 33-bus radial distribution network has been considered as the test system. The bus connections have been shown below in the figure 3.3. The bus data and line data for the system are given in appendix – a.

![Figure 3.3 IEEE 33- Bus RDS test system](image)

Figure 3.3 IEEE 33- Bus RDS test system

4. Results and Discussions

4.1 Loss Sensitivity Index (LSI) Method

In this method power losses sensitivities were calculated at all buses according to the equations given in previous chapter. The bus with minimum LSI was considered as optimal location for DG allocation. DG size was tested in the range of 0.5 MW to 5 MW with the step size of 0.5. Optimal size of DG came out be 2.5 MW. A loss reduction of approximately 48% was achieved using this method. The voltage profiles in base case i.e. without DG and after DG placement have been shown below.

![Figure 4.1: LSI plot for 33 bus RDS](image)

**Figure 4.1: LSI plot for 33 bus RDS**

![Figure 4.2: Voltage profiles in LSI method](image)

**Figure 4.2: Voltage profiles in LSI method**

4.2 Voltage Stability Index (VSI) Method

Bus number 18 was gave the minimum VSI. Therefore optimal location for DG allocation was chosen at this bus. In this method DG sizes were taken in step size of 0.5 MVA starting from 0.5 MVA up to 4 MVA at different power factors of upf, 0.9 lagging, 0.85 lagging and 0.8 lagging. Voltage sensitivity indices of different buses have been shown in the table below.

![Table 4.1: DG sizes tested in VSI method](image)

**Table 4.1: DG sizes tested in VSI method**
Table 4.2: Power loss for tested DGs

<table>
<thead>
<tr>
<th>upf</th>
<th>0.9 lag</th>
<th>0.85 lag</th>
<th>0.8 lag</th>
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<td>0.156</td>
<td>0.1592</td>
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<td>0.1638</td>
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</table>
It can be said that there is a scope for future work in this paper after the conclusions drawn from the work. These possibilities have been listed as follows: (1) The impact of both leading and lagging power factors on minimizing power losses during DG allocation can be discussed in the future work. (2) Testing of these methods on large bus systems and real time test systems is a possibility. (3) Despite the power loss reduction was appreciable in loss sensitivity index method; still there is possibility for development of better method for sizing of DG.

REFERENCES


Appendix-a

Base kV= 12.66 and Base MVA= 0.1.
Tie switches = 21-8; 9-15; 12-22; 18-33; 25-19.
### Line-data and Load-data for 33 bus RDS

<table>
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<th>Branch Number</th>
<th>Bus (From)</th>
<th>Bus (To)</th>
<th>R (ohm)</th>
<th>X (ohm)</th>
<th>P_L(kW)</th>
<th>Q_L(kVar)</th>
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