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PROPAGATION OF ELECTROMAGNETIC WAVES IN A SINGLE LAYER STRUCTURE

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Interaction of electromagnetic waves with matter should be described by quantum-mechanics, using classical electromagnetic field theory phenomenon for reflection and refraction may be expedite. From calculations it may be seen that expressions for transmittivitty and reflectivity T+R = 1 is consistent with law of conservation of energy.

Key Words : Electromagnetic Waves, Reflection, Refraction, Transmittivity, Reflectivity.

1. INTRODUCTION

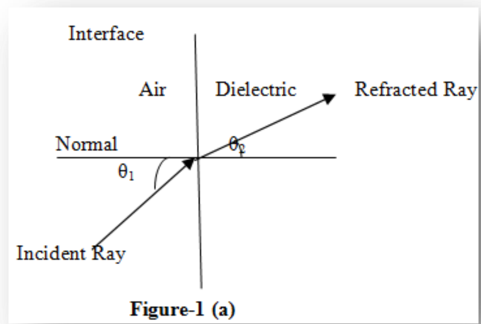
Electric charges generate electric fields and electric currents generate magnetic fields. An electric field varying in time (for example the electric field due to a moving charge) produces a magnetic field varying in space and time; this changing magnetic field produces an electric field varying in space and time; and so on. This mutual generation of electric and magnetic fields result in the propagation of the electromagnetic wave moving with velocity $v = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} =$ 2.9979×10^8 m/s in vacuum. Although the interaction of electromagnetic waves with matter should be described microscopically by quantum mechanics, it is possible to describe many phenomena such as reflection and refraction of electromagnetic waves using the classical Maxwell's electromagnetic field theory.

If a plane wave from one medium enters a different dielectric medium, it is partly reflected and partly transmitted [1], [2] when absorption is negligible. Dielectric materials have positive permittivity (ε) and positive permeability (μ). The permittivity of a material determines its response to an electric field while permeability determines its response to a magnetic field [3]. These two parameters together determine the reaction of the material to electromagnetic radiation.

Meta-materials (or negative refractive index materials) have negative permittivity, negative permeability and negative refractive [4] index $n = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0 \mu_0}}$ in a particular frequency range. Meta-materials are artificial structures which can be designed to exhibit negative permittivity and negative permeability in some particular frequency range. [5], [6] Snell's law gives the relation between angle of incidence θ_1 and angle of refraction θ_2 at an interface separating two media. It is written as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

This law applies equally well for meta- materials. Figure-1(a) & 1(b) shows the comparison between refraction in a dielectric and the refraction in a meta-material [7].



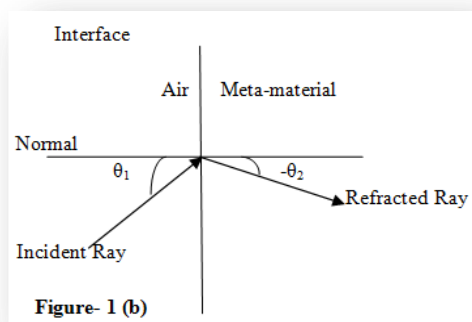


Figure-1: Comparison between Refraction in a Dielectric and Refraction in a Meta-Material

It may be noted that in a meta-material the refracted ray remains on the same side of the normal as the incident ray. In the present investigation Maxwell's electromagnetic field theory is applied to derive closed expressions analytically for the reflected power R and transmitted power T for a multilayer structure consisting of a pair of meta-material and dielectric layer inserted between two semi-infinite dielectric media.

The formulation of the problem is described in section -2; and results are discussed in section-3.

2. FORMULATION

Let us consider the reflection (and transmission) of an electromagnetic wave incident normally on a films, which thickness d_2 (having refractive index n_2) inserted between two semi-infinite dielectric media of refractive indices n_1 and n_3 [8] [9], as shown in the following figure – 2.

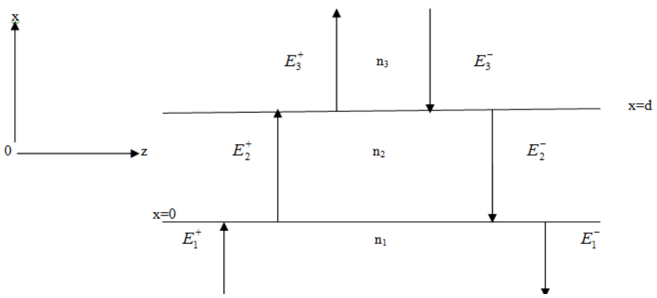


Figure – 2: Refraction and Transmission of an Electromagnetic Waves of Amplitude E_1^+

This figure shows the reflection and transmission of an electromagnetic wave of amplitude E_1^+ travelling in a dielectric medium [10] of refractive index n_1 and then incident on a film of refractive indices n_2 for a y- polarized wave, the electric fields in the three regions may be written as-

For $x < 0$,

$$\vec{E} = \hat{J}\left[E_1^+ e^{(-ik_1x)} + E_1^- e^{(ik_1x)}\right] \quad \text{---------------------- (1)}$$

For $0 < x < d_2$

$$\vec{E} = \hat{J}\left[E_2^+ e^{(-ik_2x)} + E_2^- e^{(ik_2x)}\right] \overrightarrow{E} \quad \text{-----------------------(2)}$$

For $d_2 < x < (d_2 + d_3)$,

$$\vec{E} = \hat{J}\left[E_3^{+e^{(ik_3d_2)}} e^{(-ik_3x)} + E_3^- e^{(-ik_3d_2)} e^{(ik_3x)}\right] \quad \text{---------------------- (3)}$$

Where, $k_i = (2\pi/\lambda_0)n_i$; i = 1, 2, 3

n_1, n_2 and n_3 are the refractive indices in the respective regions.

The time dependence $e^{i\omega t}$ is suppressed and the superscripts (+) and (–) correspond to waves propagating in the +x and –x directions respectively.

The corresponding magnetic fields can be calculated using the following equation

$$\vec{H} = \left(\frac{1}{\omega\mu}\right)\vec{k} \times \vec{E} \quad \text{----------------------- (5)}$$

This equation is easily derived from the Maxwell's equation $\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ and noting the fact that $\vec{E}$ and $\vec{H}$ satisfy the scalar wave equation.

Continuity of $\vec{E}$ and $\vec{H}$ at $x = 0$ gives

$$\begin{bmatrix} E_1^+ \\ E_1^- \end{bmatrix} = \frac{1}{t_1}\begin{bmatrix} 1 & r_1 \\ r_1 & 1 \end{bmatrix}\begin{bmatrix} E_2^+ \\ E_2^- \end{bmatrix} \quad \text{----------------------- (6)}$$

Where,

$$t_1 = \frac{2n_1}{n_1+n_2}$$

and

$$r_1 = \frac{n_1-n_2}{n_1+n_2}$$

Continuity at $x = d_2$ gives

$$\begin{bmatrix} E_2^+ \\ E_2^- \end{bmatrix} = \frac{1}{t_2}\begin{bmatrix} e^{i\delta_2} & r_2e^{i\delta_2} \\ r_2e^{-i\delta_2} & e^{-i\delta_2} \end{bmatrix}\begin{bmatrix} E_3^+ \\ E_3^- \end{bmatrix} \quad \text{---------------------- (7)}$$

Where,

$$t_2 = \frac{2n_2}{n_2+n_3},$$

$$r_2 = \frac{n_2-n_3}{n_2+n_3},$$

and

$$\delta_2 = k_2d_2 = k_0n_2d_2 = \frac{2\pi}{\lambda_0} n_2d_2$$

For oblique incidence δ_2 is redefined as

$$\delta_2 = \frac{2\pi}{\lambda_0} n_2d_2 \cos\theta_2 = \frac{2\pi}{\lambda_0} n_2d_2\sqrt{n_2^2 - n_1^2 \sin^2\theta_1}$$

Combining the equation (6) and (7), we get

$$\begin{bmatrix} E_1^+ \\ E_1^- \end{bmatrix} = \frac{1}{t_1t_2}\begin{bmatrix} 1 & r_1 \\ r_1 & 1 \end{bmatrix}\begin{bmatrix} e^{i\delta_2} & r_2e^{i\delta_2} \\ r_2e^{-i\delta_2} & e^{-i\delta_2} \end{bmatrix}\begin{bmatrix} E_3^+ \\ E_3^- \end{bmatrix}$$

or

$$\begin{bmatrix} E_1^+ \\ E_1^- \end{bmatrix} = \frac{1}{t_1t_2}\begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{bmatrix}\begin{bmatrix} E_3^+ \\ E_3^- \end{bmatrix} \quad \text{----------------------- (8)}$$

Where,

$$\alpha_1 = \frac{1}{t_1t_2}e^{i\delta_2} + r_1r_2e^{-i\delta_2}$$

$$\alpha_2 = \frac{1}{t_1t_2}r_1e^{i\delta_2} + r_2e^{-i\delta_2}$$

$$\beta_2 = \frac{1}{t_1t_2}r_2e^{i\delta_2} + r_1e^{-i\delta_2}$$

$$\beta_2 = \frac{1}{t_1 t_2} r_1 r_2 e^{i\delta_2} + e^{-i\delta_2}$$

Now, in the fourth medium, there will not be any reflected wave and as such $E_3^- = 0$ Hence equation (7) gives

$$E_1^+ = aE_3^+ \qquad \text{and} \qquad E_1^- = cE_3^+$$

Hence, amplitude reflection coefficient (r) is obtained as,

$$r = \frac{E_1^-}{E_1^+} = \frac{a}{c} \quad \text{-----------------------(9)}$$

And amplitude transmission coefficient (t) is obtained as,

$$t = \frac{E_3^+}{E_1^+} = \frac{1}{a}$$

The reflectivity R and transmittivity T are given by $R = |r|^2$ and

$$T = \frac{n_3}{n_1} |t|^2 \quad \text{----------------------(10)}$$

Final expressions obtained for R and T are as follows:-

$$R = \frac{-A_1 + B_1 + C_1 \cos 2\delta_2}{B_1 + C_1 \cos 2\delta_2} \quad \text{--------------------- (11)}$$

and

$$T = \frac{A_1}{B_1 + C_1 \cos 2\delta_2} \quad \text{--------------------- (12)}$$

Where, $\delta_2 = k_0 d_2 (n_2^2 - n_1^2 \sin^2 \theta_1)^{1/2}$

In equations (11) and (12), the values of A_1, B_1, and C_1 are as follows:

$$A_1 = 16 n_1 n_2^2 n_3$$

$$B_1 = (n_1 + n_2)^2 (n_2 + n_3)^2 + (n_1 - n_2)^2 (n_2 - n_3)^2$$

$$C_1 = 2(n_1 - n_2)^2 (n_2 - n_3)^2$$

$$T + R = \frac{A_1}{B_1 + C_1 \cos 2\delta_2} + \frac{-A_1 + B_1 + C_1 \cos 2\delta_2}{B_1 + C_1 \cos 2\delta_2}$$

$$= \frac{A_1 - A_1 + B_1 + C_1 \cos 2\delta_2}{B_1 + C_1 \cos 2\delta_2}$$

$$= \frac{B_1 + C_1 \cos 2\delta_2}{B_1 + C_1 \cos 2\delta_2}$$

$$T + R = 1$$

These expressions immediately give

$$T + R = 1$$

We can obtain the expression for transmittivity for the case of one film if we set $n_2 = n_3$ in equation (12). Thus for the case of one film, we get from equation (12)

$$T = \frac{8 n_1 n_2}{(n_1 + n_2)^2} \quad \text{----------------------(13)}$$

This expression is same as that derived in [1].

Expression for transmittivity given by equation (12) is general and applicable for a dielectric or meta-material. When one of the film is a meta-material film and its refractive index n_3 is numerically equal to the refractive index n_2 of the dielectric film i.e., $n_3 = -n_2$. Then equation (13) gives on Simplification, T = 1.

3. RESULT

Expressions for transmittivity T and reflectivity immediately give $T + R = 1$, which is consistent with the law of conservation of energy. The expressions derived in section-2 are general and applicable for a dielectric or meta-material.
The derived expressions immediately give the result that the reflected power is zero when the semi-infinite dielectric media have the same refractive index and the layer have the width d, also the refractive indices of meta-material and the dielectric are numerically equal.

The condition of zero reflectance becomes necessary in many applications. For example, in an interference filter we require a small region of wavelength so that when electromagnetic waves of wide frequency range are incident, the output will have a very small spread in wavelength.

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